Optimization of Buckling and Yield Strengths of Laminated Composites

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Theme

FIBER-reinforced laminates possess two important features highly desirable in aerospace structural applications. These are: 1) the high stiffness to weight ratio, and 2) the anisotropic property which can be tailored through the variation of fiber orientations and the stacking sequence. The latter feature provides the structural designer with an additional dimension in the constraint space that could lead to a substantial change in the design procedure. Furthermore, as a result of the additional variables, the structural optimization becomes an integral part of a good design work involving laminates of advanced composite materials.

Under in-plane loads, a laminated plate can fail as a result of structural instability (buckling) or material yielding. In this report we are concerned with the optimum fiber orientation for a set of combined loadings. The laminates considered are the angle-ply laminates with symmetric layups. A direct search technique is employed to solve the optimization problems.

Although the optimum fiber directions are determined independently according to the buckling and the strength constraints, the results can be easily placed in the respective regions where they are meaningful. The buckling loads for a plate depend on the geometry and size of the structure, while the strength criterion usually depends only on the state of stress at a point. It is conceivable that, for practical purposes, a marginal size for the laminated plate can be defined so that when the plate is larger than this size the laminate should be optimized according to the buckling criterion. For a plate size smaller than the marginal size, the strength criterion should be followed. This design procedure is discussed in some detail in this report.

Contents

Laminated plates consisting of a finite number of layers of fiber-reinforced materials are considered. The layer properties for all the lamina are identical except the fiber orientations. A plate theory[1] that neglects the transverse shear deformation is employed for the analysis.

We consider a simply supported, symmetrically laminated rectangular plate under the action of combined in-plane normal loadings N_1, N_2, and in-plane shear N_6. A solution for the critical buckling load can be obtained from the energy principle.

$$V = W - W_1 = \text{stationary value} \tag{1}$$

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where V is the total potential of the plate, W is the total strain energy, and W_1 is the work done by the applied forces.

The Ritz method is used to find an approximate solution to the variational problem $\delta V = 0$. It is assumed that

$$u_3 = \sum_{i=1}^{m} \sum_{j=1}^{m} W_{ij} \sin \frac{i\pi x_1}{a} \sin \frac{j\pi x_2}{b} \tag{2}$$

where u_3 is the transverse displacement, and a and b are the plate-dimensions in the x_1- and x_2-directions, respectively, (Fig. 1). The expression given by Eq. (2) does not satisfy automatically the boundary conditions for the simply supported plate. This can be corrected to some extent by adding the work done by the unbalanced edge moments to W_1. A detailed discussion on this type of correction was given by Ashton.[2]

For convenience, the applied in-plane loads are expressed in terms of the load ratios

$$L_i = N_i / P \quad i = 1,2,6 \tag{3}$$

where P is a referential load. We define the buckling coefficient K_{cr} as

$$K_{cr} = a^2 P_{cr} / E_L h^3 \tag{4}$$

where E_L is the Young's modulus in the fiber direction, h is the plate thickness and P_{cr} is the critical referential load at which instability occurs. The buckling load is found to satisfy the generalized eigenvalue problem

$$[F] \ [X] = \lambda \ [H] \ [X] \tag{5}$$

where the eigenvalue $\lambda = 1/K_{cr}$, $[F]$ is a symmetric matrix, and $[H]$ is symmetric and positive definite.

To obtain specific results, we consider a 20-ply glass-epoxy laminated plate with angle-ply symmetric layup. The only variable is the angle θ between the x_1-axis and the fiber directions. The optimization of the buckling load is equivalent to minimizing the performance index

$$IP = -\lambda(\theta) \tag{6}$$

subject to

$$[F(\theta)] \ [X] = \lambda(\theta) \ [H(\theta)] \ [X] \tag{7}$$

The golden-section search is used to find the maximum buckling coefficient and the corresponding optimum fiber-

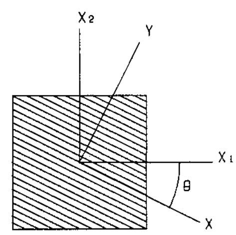

Fig. 1 Material coordinates (x,y) and plate coordinates (X_1, X_2).

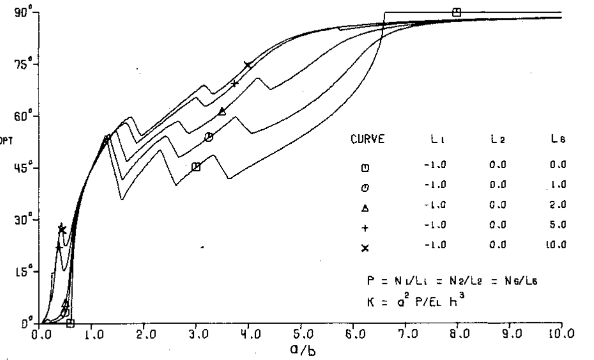

Fig. 2 Optimum fiber orientation vs aspect ratio for one-way compression combined with shear loadings.

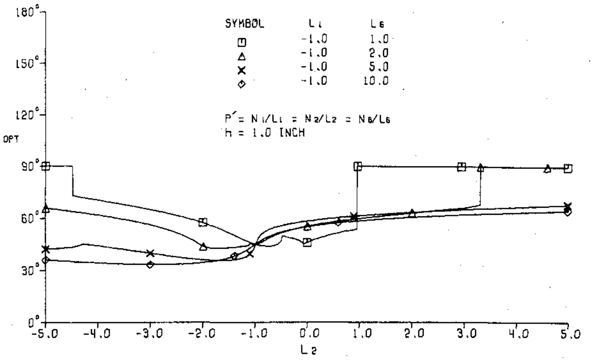

Fig. 3 Optimum fiber orientation for maximum yield strength of a laminate under combined loads.

orientation. In Fig. 2 the optimum fiber orientations in the laminate subjected to a compressive load N_1 in the X_1 direction and various shearing loads N_6 are shown. The material constants of the glass-epoxy composite are chosen to be

$$E_L = 7.8 \times 10^6 \text{psi}, E_T = 2.60 \times 10^6 \text{psi},$$
$$G_{LT} = 1.25 \times 10^6 \text{psi}, \nu_{LT} = 0.25$$

Several cases of the strength of quasi-homogeneous composites have been investigated by Tsai and Azzi.[3] The problem was to determine the state of the combined stresses which cause failures of the composite. In this report, we are interested in the optimum fiber-orientation at which the laminate has the maximum yield strength. The yield criterion according to the theory of maximum work for the orthotropic material is given by

$$\left(\frac{\sigma_x}{X}\right)^2 + \left(\frac{\sigma_y}{Y}\right)^2 - \frac{\sigma_x}{X}\frac{\sigma_y}{X} + \left(\frac{\tau_{xy}}{S}\right)^2 = 1 \qquad (8)$$

where X, Y, and S are the longitudinal tensile strength, trasverse tensile strength and the shear strength, respectively, the coordinates x and y are parallel to an perpendicular to the fibers, respectively. If any of the normal stresses is negative, the corresponding compressive strength X' or Y' must be used. By denoting the referential load P' for the yield strength in the same form as P given by Eq. (3), the stresses in the kth layer can be expressed as

$$\sigma_x^{(k)} = P' A_0, \ \sigma_y^{(k)} = P' B_0, \ \tau_{xy}^{(k)} = P' C_0 \qquad (9)$$

where A_o, B_o, and C_o are functions of the material constants, the load ratios L_1, L_2, and L_6, and the fiber orientation θ. The optimum referential load is obtained by minimizing the performance index

$$IP = -P'(\theta) \qquad (10)$$

with L_i given.

Computations are carried out for a 20-ply glass-epoxy interspersed laminate for which

$$X = 150 \times 10^3 \text{psi}, \ Y = 4 \times 10^3 \text{psi}, \ S = 8 \times 10^3 \text{psi}$$

$$X' = -150 \times 10^3 \text{psi}, \ Y' = -20 \times 10^3 \text{psi}$$

Figure 3 shows the variation of the optimum fiber orientation in the plate under combined loadings. A clear characteristic of these results is that the optimum fiber orientation can undergo a drastic change with a small variation of the loading. Such abrupt change was also found in the strength optimization for cylindrical shells of laminated composites.[4] The reason is that there exist multiple local maximum referential loads, and, as a result, a small change of loading can lead to a switch of the global maximum from one local maximum to the other.

For a laminated plate under compressive in-plane loadings it can either buckle or yield, depending on the type of loading and more importantly the size of the plate. The question concerning the choice between the criteria for the optimum buckling strength and the optimum yield strength then naturally arises. To answer this, it should be noted that the optimization for yield strength does not depend on the size of the laminate and the aspect ratio, whereas the buckling strength is greatly affected by the dimensions of the plate. It is conceivable that there exists a "marginal" size of the plate above which $p'_{opt} > P_{opt}$, and the optimum fiber direction must be chosen according to the consideration of the buckling strength. On the other hand, below this marginal size, we have $P_{opt} > P'_{opt}$ and the fibers must be oriented according to the yield strength in order to achieve the strongest plate for the given load and the aspect ratio. This marginal size can be obtained from Eq. (4) as

$$a_{\text{marg}} = (K_{\text{opt}} E_L \ h / P'_{\text{opt}})^{1/2} \qquad (11)$$

where P'_{opt} is determined from the yield strength and K_{opt} is the optimum buckling coefficient. Thus, under a given loading condition and an aspect ratio, the consideration of yield strength governs the optimum design criterion if the plate lies within the marginal size, whereas the buckling strength should be taken as the criterion if the size of the plate exceeds this limit. However, one should realize that the marginal size as given by Eq. (11) is only approximte in nature and that it yields accurate guidance if the plate-size is substantially larger or smaller than a_{morg}. For plate sizes that are in the neighborhood of this size, both criteria should be tested and compared.

References

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